(Even hole, triangle)-free graphs revisited

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Summary

Even hole-free graphs

- No structure theorem is known for even-hole-free graphs.

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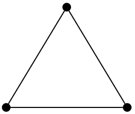
Subclass (even hole, triangle)-free [Conforti et al., '00]



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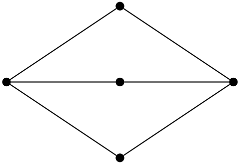
Triangle-free odd signable graphs [Conforti et al., '00].

- Triangle-free odd signables graphs are free of:
  - Even-wheel;
  - Theta;
  - Triangle.

What are we excluding?



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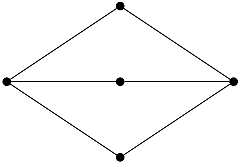

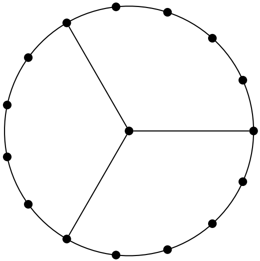

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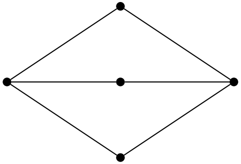

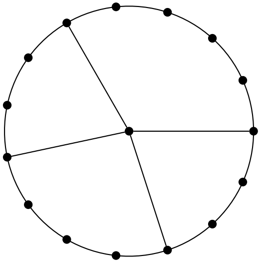

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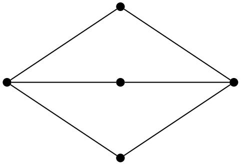

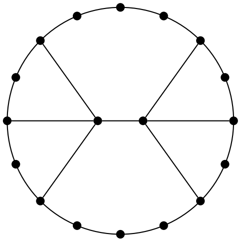

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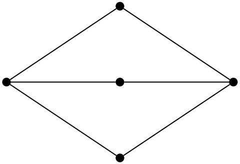

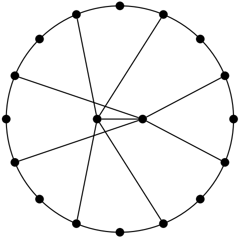

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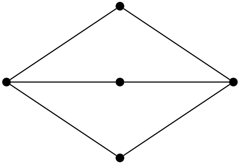

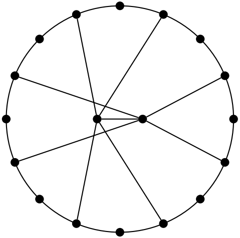

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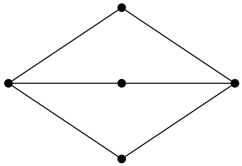

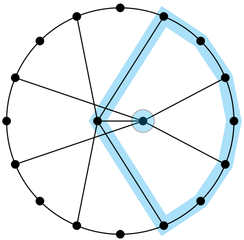

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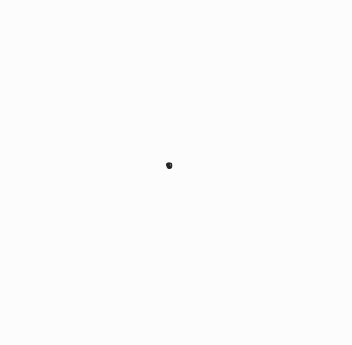

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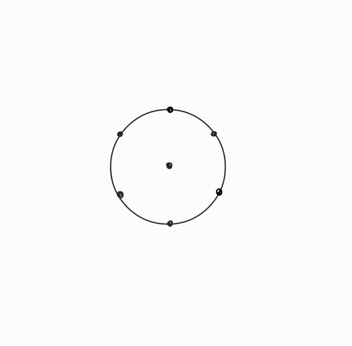

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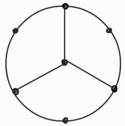

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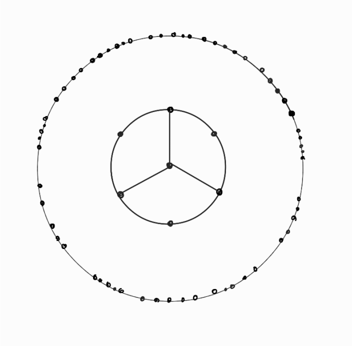

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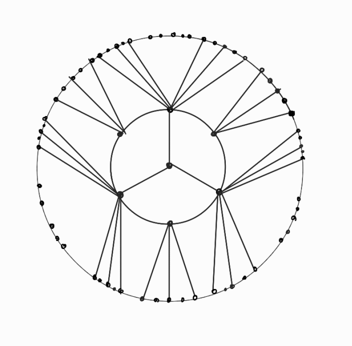

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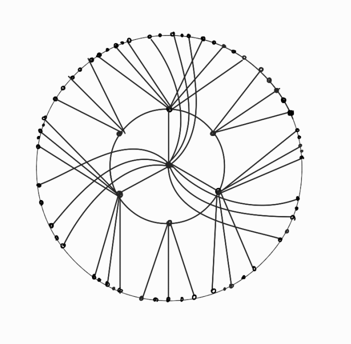

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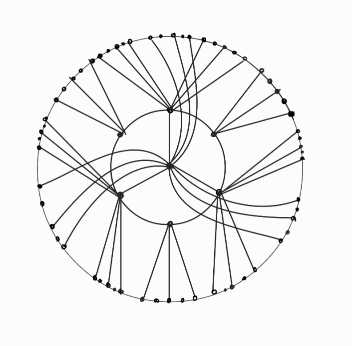


tw$\leq f$(number of layers)?

Our main result

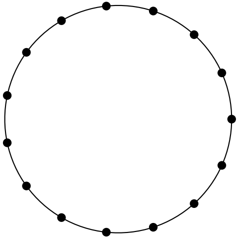
Theorem [M., Trotignon, '26]

If G is a (theta, triangle, wac)-free graph, then G is basic or G has a clique separator, a proper 2-separator or a proper P_3-separator.

Basic graphs

Daisy

Basic graphs
Daisy



Basic graphs
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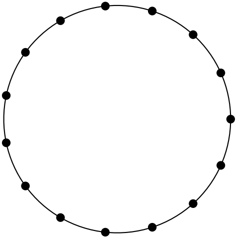
Definition

A *petal* centered in c_i is a path $P = x \dots y$ disjoint from C such that $xc_{i-1},\ yc_{i+1} \in E(G)$, $zc_i \in E(G)$ for at least one internal vertex z of P and $N_C(w) \subseteq \{c_i\}\ \forall w \in P$.



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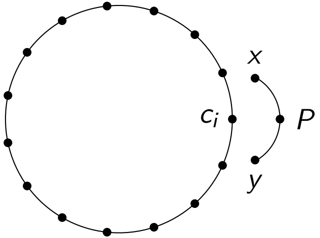

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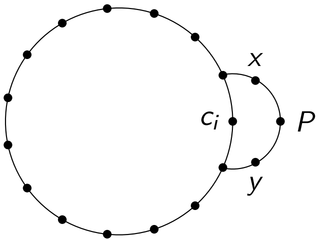

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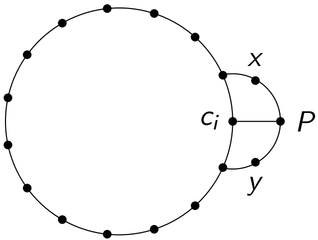

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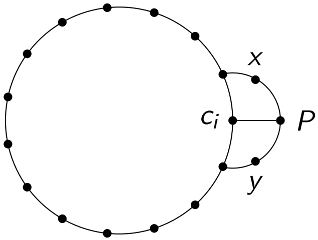

Wheel with c_i as its centre

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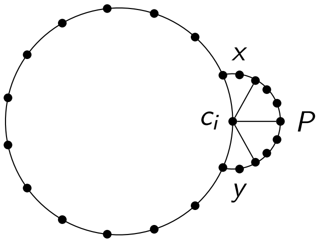

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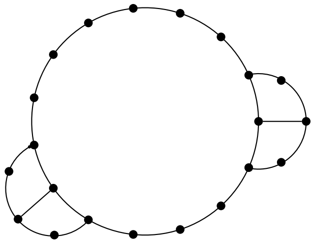

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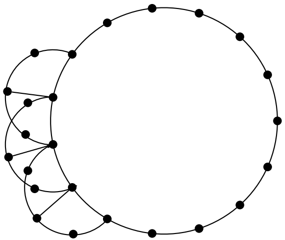

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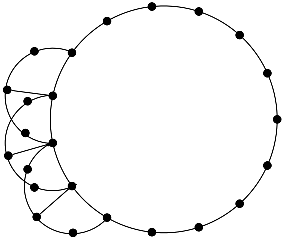
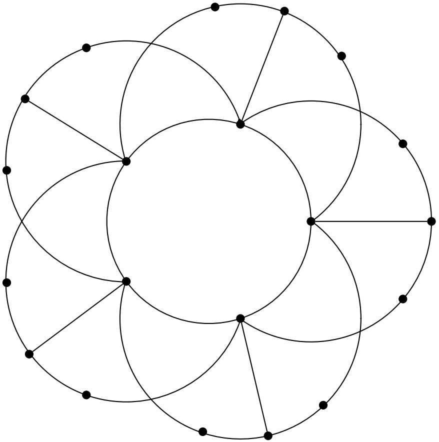


Definition

A *daisy* is a graph G formed of a hole C together with consecutive petals with respect to C.

Basic graphs
Daisy



A non-planar daisy presented in [Conforti et al., '00].

Basic graphs

Cube, K_1 and K_2

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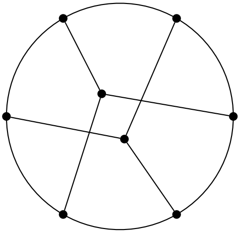
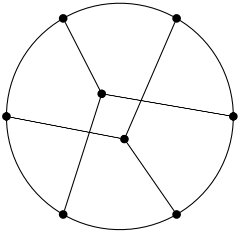
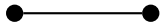




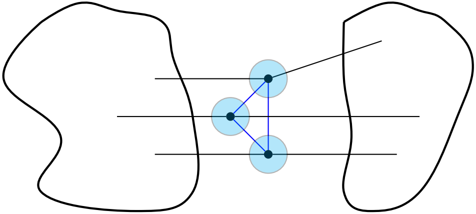


Definition

A graph is *basic* if it is isomorphic to K_1, K_2, the cube or a daisy.

Definition

A *clique separator* in a graph G is a (possibly empty) clique K such that $G \setminus K$ has at least two connected components.



Separators

Clique separator

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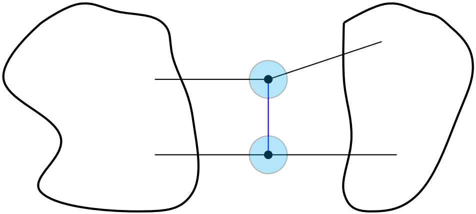


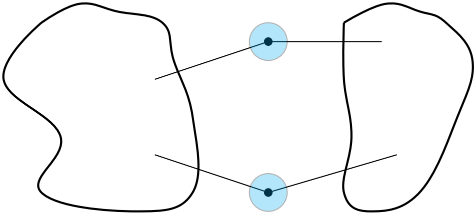
Since we work with triangle-free graphs, the only possible clique separators are K_1 and K_2.

Separators

2-separator

Definition

When a and b are non-adjacent vertices of some graph G such that $G \setminus \{a, b\}$ is not connected, we call $\{a, b\}$ a *2-separator* of G.



Definition

A 2-separator $\{a, b\}$ of G is *proper* if:

- $G \setminus \{a, b\}$ has exactly two components X and Y;
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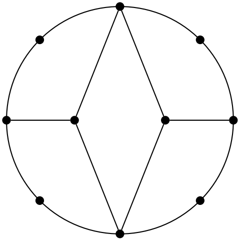

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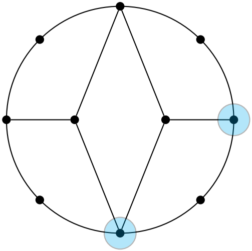

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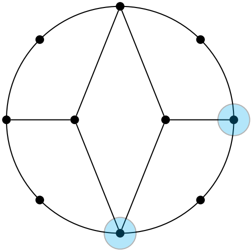

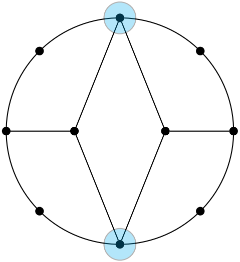

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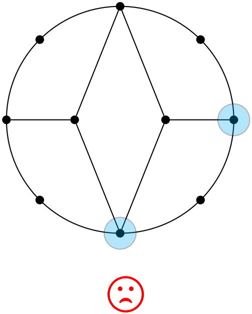

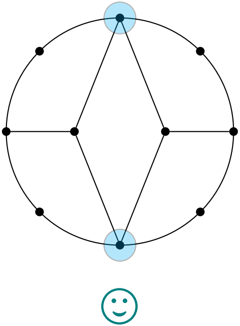

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P_3-separator

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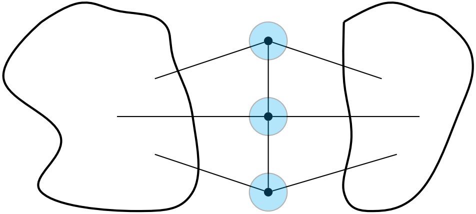
When acb is a path in some graph G such that $G \setminus acb$ is not connected, acb is a P_3-*separator* of G.



Separators

P_3-separator

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P_3-separator

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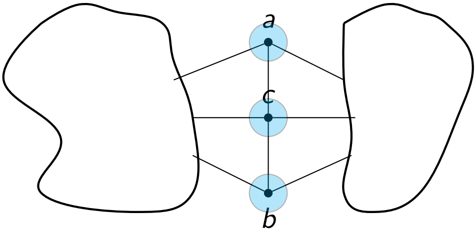
When acb is a P_3-separator, and X is a connected component of $G \setminus acb$, we say that X is *loose* if there exists an aXb-path with no internal vertex adjacent to c, and *tight* otherwise.



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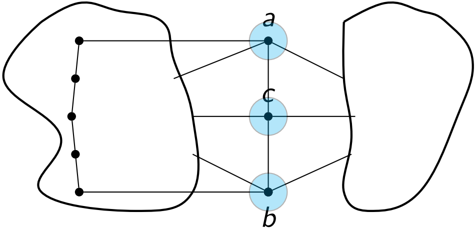

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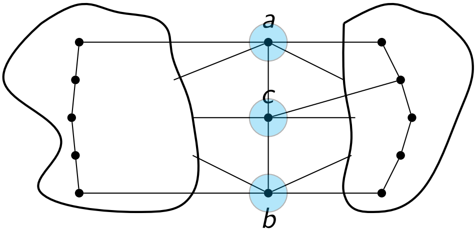

Definition

A P_3-separator acb of G is *proper* if $G \setminus acb$ has exactly two components X and Y, there exists an aXb-path and an aYb-path in G, c has neighbours in both X and Y, X is loose, Y is tight, and none of $G[X \cup \{a, b\}]$ and $G[Y \cup \{a, b\}]$ is an ab-path.

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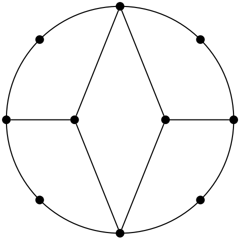

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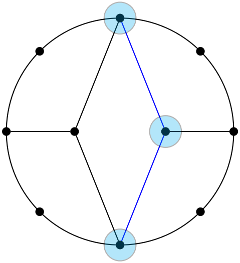

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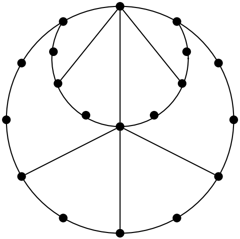

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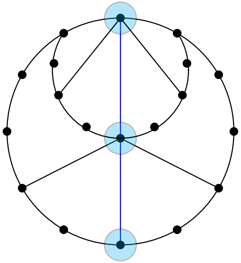

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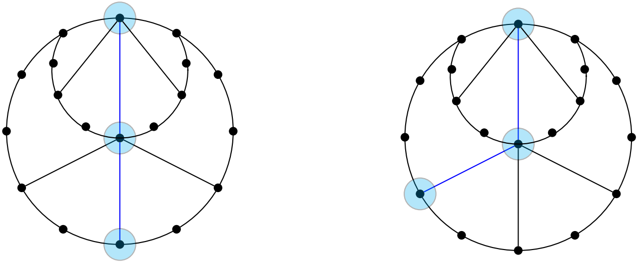

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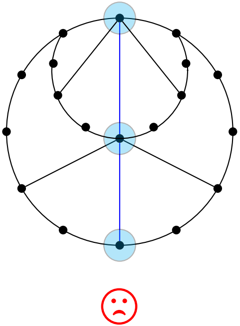

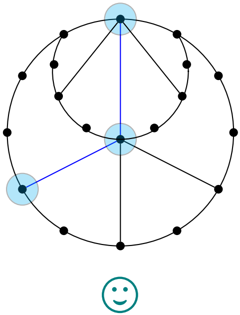

Recapitulation

Theorem [M., Trotignon, '26]

If G is a (theta, triangle, wac)-free graph, then G is basic or G has a clique separator, a proper 2-separator or a proper P_3-separator.

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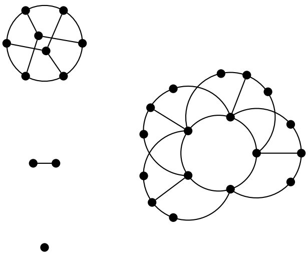

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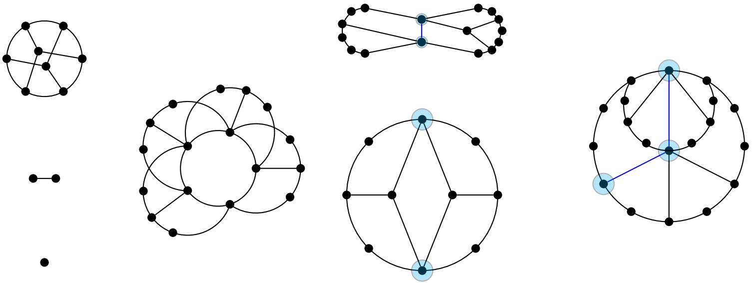

Recognition Algorithm

Theorem [M., Trotignon, '26]

There exists an algorithm that decides in time $O(|V(G)|^4|E(G)|)$ whether an input graph G is (theta, triangle, wac)-free (resp. (theta, triangle, even wheel)-free, (even hole, triangle)-free, bipartite (theta, wac)-free).

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Blocks of decomposition

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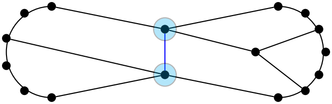

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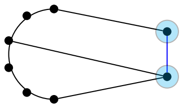

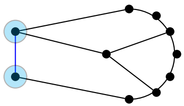

Blocks of decomposition

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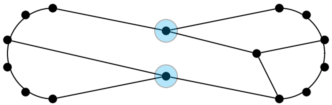

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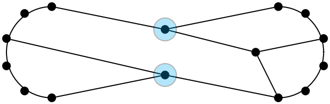

⚠ This graph has a clique separator

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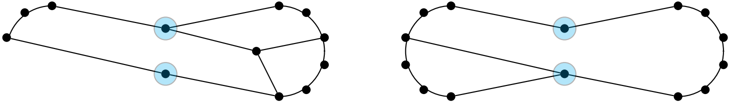

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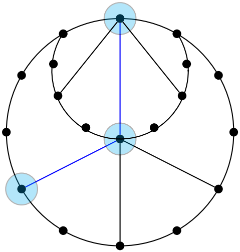

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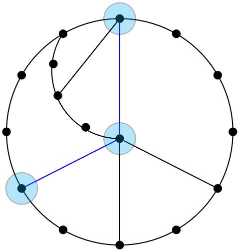

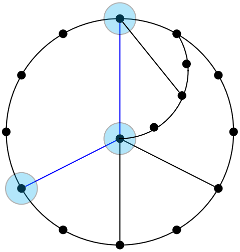

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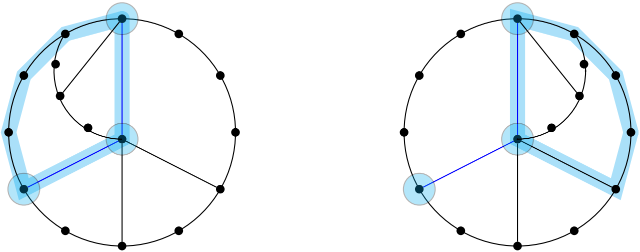

Blocks of decomposition

Lemma

G is a non-basic (theta, triangle, wac)-free graph if and only if its blocks of decomposition are (theta, triangle, wac)-free.

Decomposition tree

Definition

- Given G, T_G is a rooted tree, and its root is G itself.
- If $H \in V(T_G)$ has some clique separator, then for a minimal clique separator K, the children of H are the blocks of decomposition of H with respect to K.
- Similar for proper 2-separator and proper P_3-separator.

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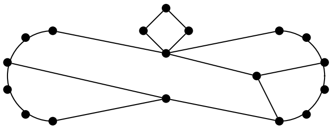
Lemma

A graph G is (theta, triangle, wac)-free (resp. (theta, triangle, even wheel)-free, (even hole, triangle)-free, bipartite (theta, wac)-free)) if and only if all the leaves of T_G are basic graphs from this class.

Decomposition tree



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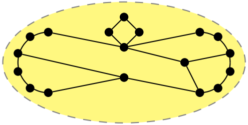

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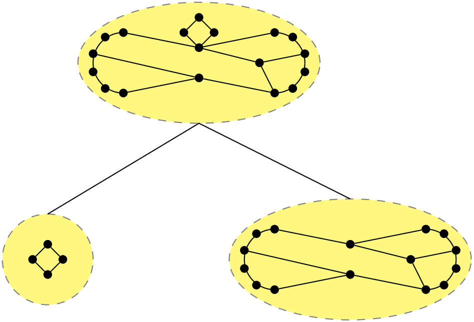

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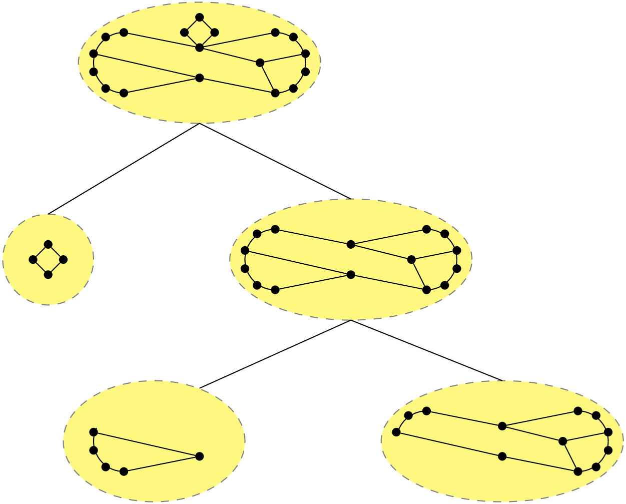


Decomposition tree

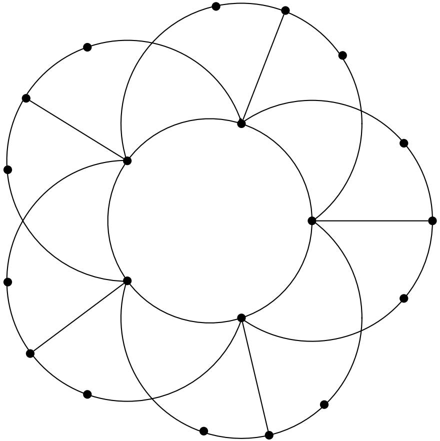
Lemma [M., Trotignon, '26]

If G is (theta, triangle, wac)-free, then T_G has at most $2|V(G)|-1$ nodes.

Planarity

Theorem [M., Trotignon, '26]

A (theta, triangle, wac)-free graph is planar if and only if it contains no full odd daisy.



Tree-decomposition

Theorem [Cameron et al., '18]

Every (theta, triangle, even wheel)-free graph has treewidth at most 5.

Theorem [M., Trotignon, '26]

A (theta, triangle, wac)-free graph has treewidth 4 if it contains a full k-daisy where $k \geq 5$, treewidth at most 2 if it is wheel-free, and treewidth 3 otherwise.

Tree-decomposition

Lemma

Let G be a graph with a clique separator, a proper 2-separator or a proper P_3-separator and let $G_1, \ldots, G_k$ be its blocks of decomposition with respect to this separator. We have $\text{tw}(G) = \max(\text{tw}(G_1), \ldots, \text{tw}(G_k))$.

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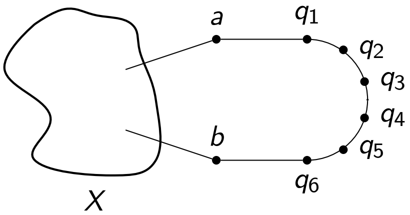

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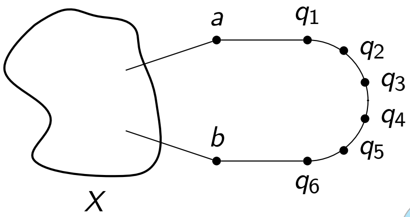

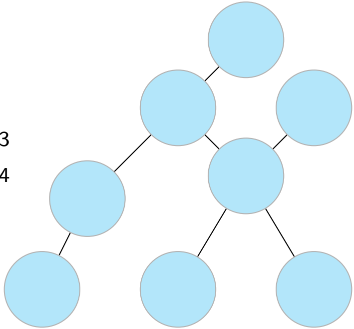

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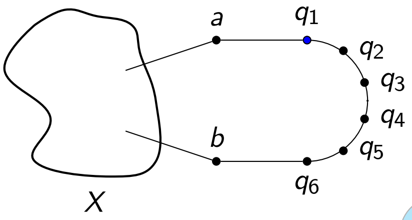

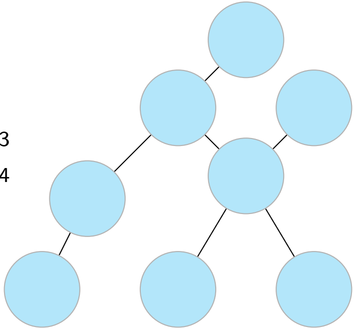

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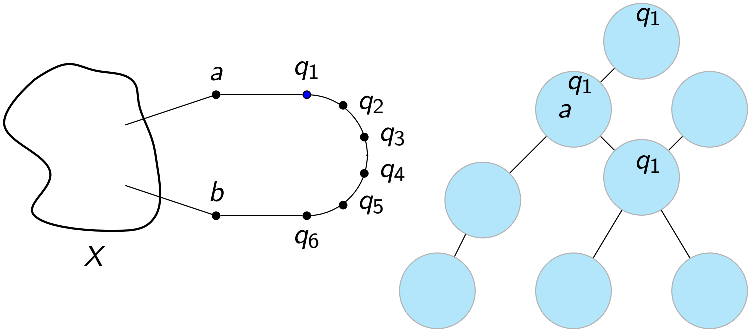

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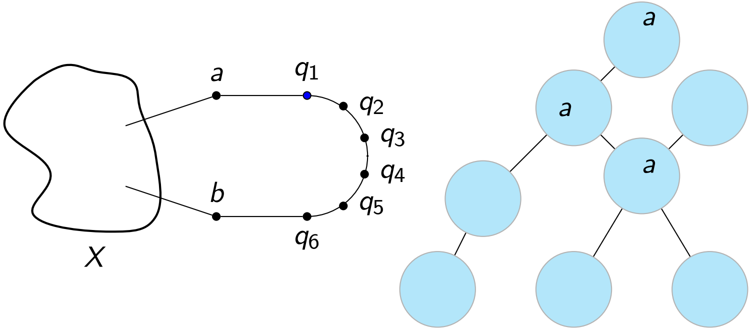

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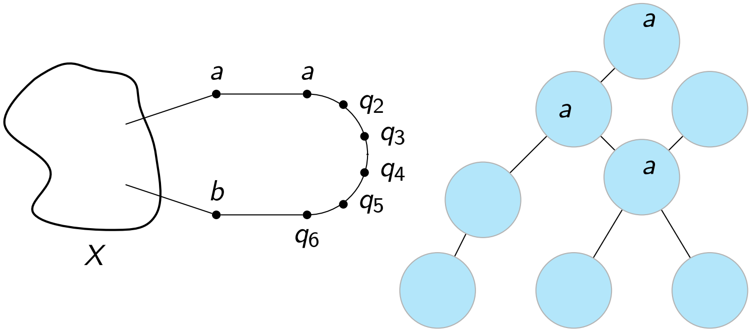

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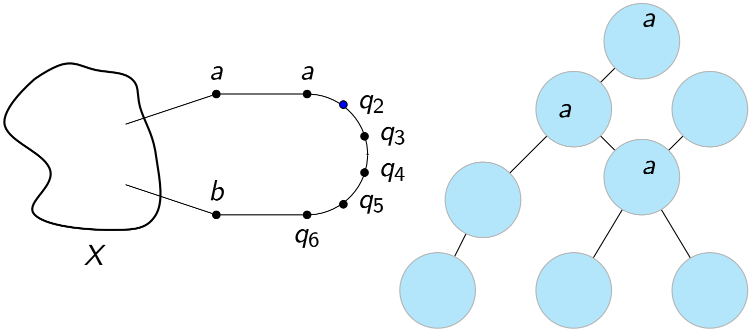

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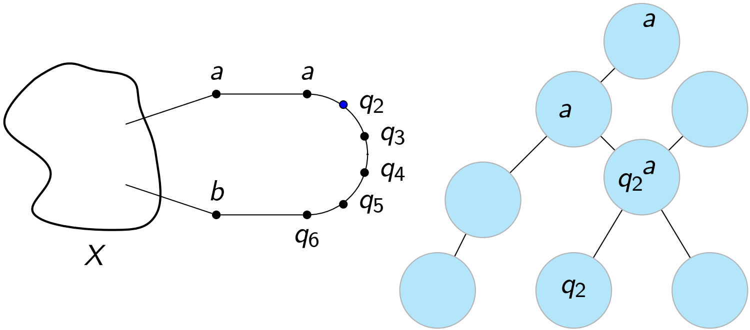

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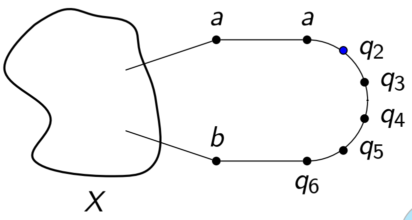

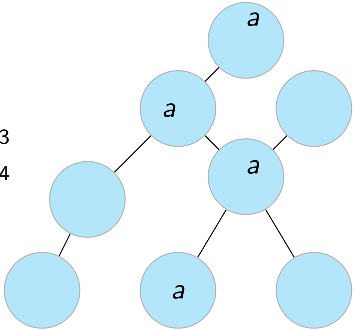

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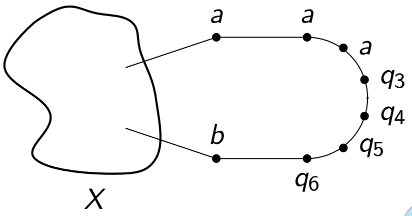

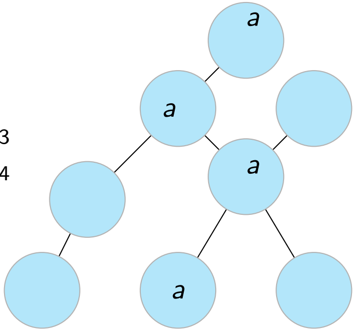

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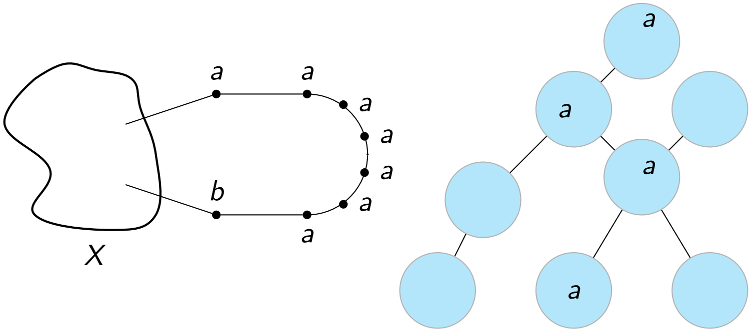

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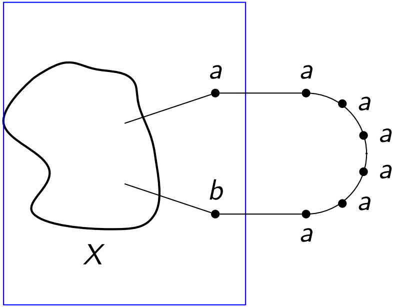

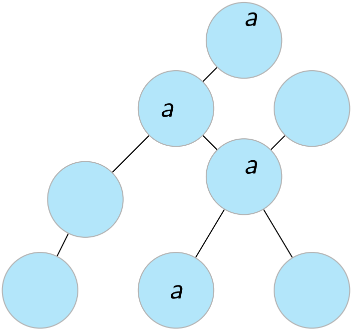

How can we attach something to a wheel?

If not attached to the center

Lemma

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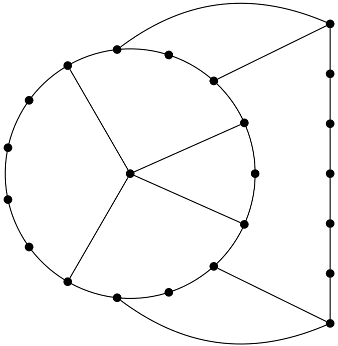

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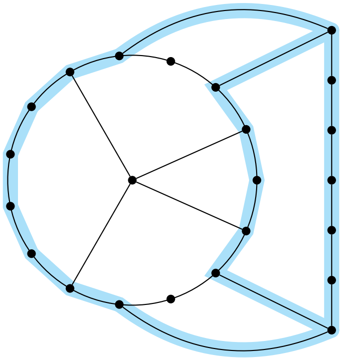

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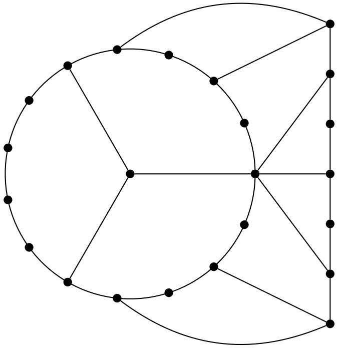

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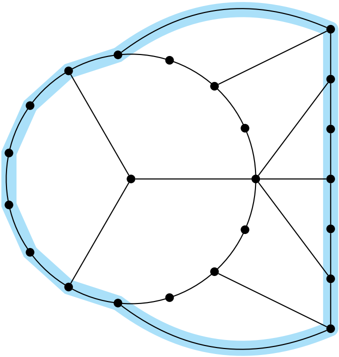

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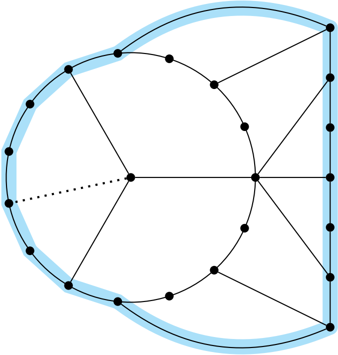

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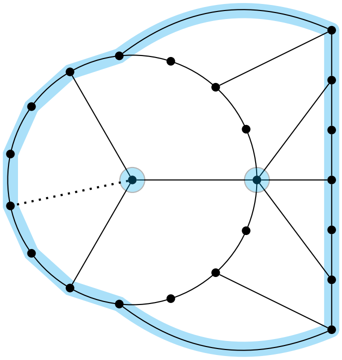

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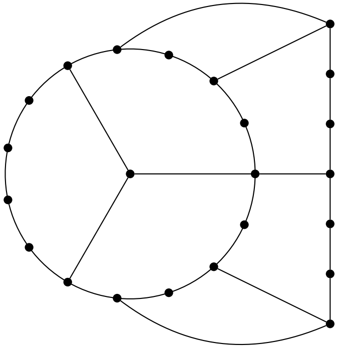

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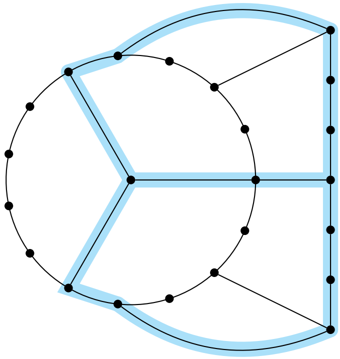

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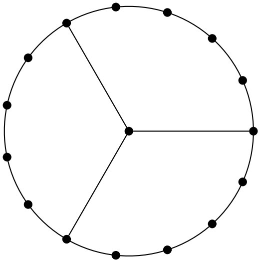

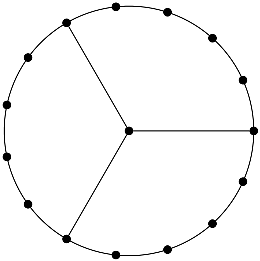

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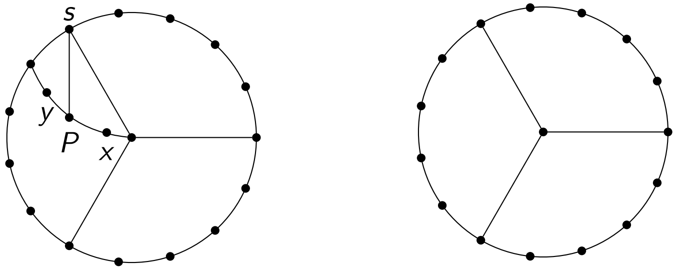

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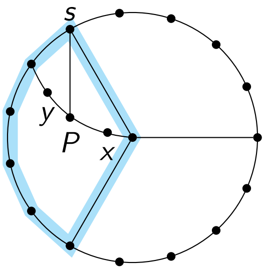

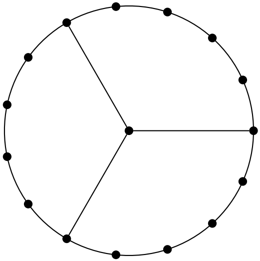

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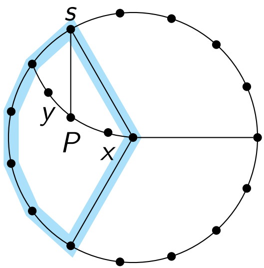

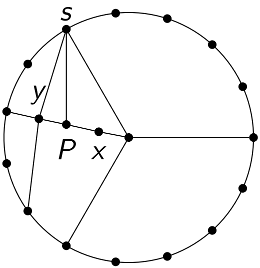

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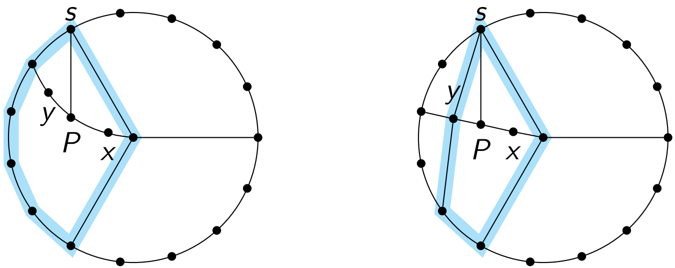

Decomposition theorem

Theorem [M., Trotignon, '26]

If G is a (theta, triangle, wac)-free graph, then G is basic or G has a clique separator, a proper 2-separator or a proper P_3-separator.

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Lemma [Radovanović, Vušković, '13]

If G is a (theta, triangle)-free graph that contains the cube, then G is isomorphic to the cube or G has a clique separator.

Open questions

- Separators for (theta, triangle, non-planar wac)-free graphs?
- A structure theorem for (theta, triangle)-free graphs? With layered wheels as a basic class?

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Thank you!